Stability of Two-Phase Vertical Flow in EOR Applications

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Abstract

This paper investigates the interfacial instability that occurs due to the displacement of reservoir oil by an injected solvent of lesser viscosity and density than oil. Instability in the form of interfacial distortions arises due to unfavorable contrasts of both density and viscosity between the injected solvent and reservoir oil. The resulting viscous and gravitational fingers lead to the formation of large channels of the injected solvent that bypass the reservoir oil and lower the sweep efficiency substantially compared to the hypothetical stable displacement. Instability characteristics are quantified in terms of the width of unstable fingers and their rate of propagation and growth. We employ a normal mode, matched asymptotic expansion method to obtain analytical expressions governing the incipient stability behavior of such flows. We show that in the case of vertically oriented injection, instability can occur at two fronts moving in opposite directions with unique characteristics such that the maximum growth rate decreases both when the mobility ratio is increased at the front end and decreased at the back end. Linear stability analysis indicates that the most important parameters governing instability behavior are the shock viscosity ratio, capillary number and the relative permeability functions. We systematically explore the influence on instability due to the variation of these parameters. We carry out high accuracy numerical simulations based on spectral methods to investigate nonlinear instability properties related to the long term development of unstable structures. Our high accuracy numerical simulations are able to resolve all the relevant length and time scales of the unstable displacement that are determined from the stability analysis. This provides a rigorous validation of the numerical simulator. The late time nonlinear behavior is marked by interactions between the fingers, similar to that for miscible displacements and the wavelength coarsening characteristics follow a linear growth in time for a wide range of the parameters.

1. Introduction

Recovery of oil from underground formations is usually enhanced by the injection of a fluid, such as water or gas, into the reservoir in order to displace oil towards production wells. Common reservoir displacement processes are water and solvent injection referred to as immiscible multiphase displacements.

Displacement processes are sensitive to the properties of the fluids involved. If the injected fluid is less viscous than oil it would likely finger through the oil, leading to a nonuniform displacement front. On the other hand, water will displace oil in a more uniform manner if the water viscosity is greater than that of the oil. The effect of density differences can lead to either gravity override or underride, and that can also affect the sweep efficiency negatively. The recovery of so-called "heavy" (i.e., high viscosity) oil by waterflooding is not common because the viscosity contrasts in such flows are considered extremely unfavorable[1].

The local displacement efficiency of immiscible displacements is generally limited by an amount of trapped oil which depends upon the wettability properties of the injected water and porous medium. Viscous fingering at very high viscosity contrasts will lead to bypassing which can further reduce the sweep efficiency. We are therefore interested in developing a fundamental understanding of the stability characteristics of immiscible reservoir flows where the unfavorable viscosity ratio is significantly large. The behavior of unstable miscible displacement processes has been studied extensively. On the other hand, the stability behavior of immiscible flows has not received comparable attention.

Chouke[2] and others examined the stability of an immiscible displacement where the two fluids are separated by a sharp interface. This approximation assumes distinct phases separated by a step discontinuity with macroscopic surface tension acting on the interface. This analysis is quite appropriate for the Hele-Shaw model. In porous media, however, one must account for a finite thickness transition zone where capillary effects are distributed over a large number of microscopic interfaces. Most importantly, flows in porous media are characterized by the simultaneous flow of two phases.

Using simple stability analysis based on a linear base profile, Hagoort[3], and later Chorin[4] highlighted the importance of simultaneous flow of multiple phases. They both used the so-called shock mobility ratio as a measure of instability. The shock mobility ratio is the ratio of the total mobility at the shock saturation to that ahead of the front. Implicit in this analysis is the assumption that the shock mobility ratio is the governing criterion for instability even though the mobility ratio across the full transition region is much larger. Chikhliwala *et al.*[5] presented numerical dispersion curves for small values of the viscosity ratio, while Yortsos and Hickernell[6] derive analytical expressions for the growth rate in the limit of small wavenumbers. Our effort extends the investigation to large viscosity ratios and flows with the density variation. We also examine the dependence of stability characteristics on the functional form of relative permeability coefficients.

When injection occurs at saturation values higher than the shock saturation the mobility ratio across the whole saturation profile is much larger than that across the shock. Hence, we investigate whether or not the shock mobility ratio is the governing stability criterion by perturbing the full range of saturations for a complete Buckley–Leverett profile with capillary dispersion. The focus of the present study is on the instability properties of large viscosity ratio displacements which can be quite different from those at small viscosity ratios. We find that for moderate to large viscosity ratio values the maximum growth rate of the instability scales linearly with the viscosity ratio while both the wavenumber corresponding to the maximum growth rate and the cutoff wavenumber scale with the square root of the viscosity ratio. We also address the important issue of a mobile wetting-phase initial saturation to show that the presence of simultaneously flowing phases, both ahead of and behind the transition region, can lead to relatively less unstable behavior than the case of purely nonwetting phase flowing ahead of the shock. We have also observed that the details of the relative permeability functions can have an important influence on the instability mechanism.

Stability of miscible flows in the presence of competing gravitational and viscous effects was initially carried out by Dumoré[7]. A detailed linear stability analysis was later carried out by Manickam and Homsy[8]. The latter authors establish equivalency conditions between upwards and downward flows as well as point out similarities between gravitationally unstable vertical displacements and viscously unstable horizontal displacements with nonmonotonic viscosity profiles. Although immiscible, gravitationally unstable displacements have been studied by a number of authors a fundamental understanding of the linear stability behavior is lacking. In the case of immiscible displacements, the presence of simultaneously flowing phases precludes the use of first-order theoretical predictions, appropriate for Hele-Shaw flows to infer the instability length scales in porous media We address the problem of gravitationally stable and unstable immiscible displacements using the fractional flow formulation. The growth rates and the length scales are computed for a range of gravity and viscosity parameter combinations. Gravitational stabilization of flows with unfavorable viscosity contrast and viscous stabilization of gravitationally unstable flows is also discussed. Finally, we develop a marginal stability curve in the viscosity ratio and gravity number parameter space[9].

2. Unstable Displacements

The influence of the viscosity ratio on the stability of the displacement process is shown in Figure 1 which plots the growth rate vs wavenumber curves for a typical problem, Figure 6(a) shows that the growth rate is positive when the mobility ratio across the shock is > 1 and is negative otherwise. The maximum growth rate as well as the cutoff and the most dangerous wavenumbers increase with an increase in the viscosity ratio *M*. Figure 6(b) shows that very large growth rates are associated with large values of *M*.

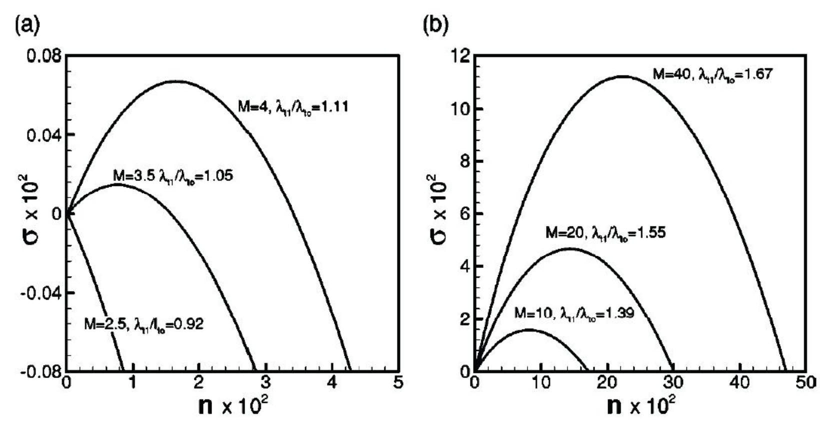

Figure 1. Growth rate vs wavenumber dispersion relation showing the influence of the viscosity ratio *M*. (a) Small values of *M*. (b) Large *M* values.

Figure 2 shows a distinctly different behavior of the maximum growth rate at small values of the viscosity ratio as compared to larger values. Both the maximum growth rate as well as the cutoff and most dangerous wavenumbers increase rapidly for small values of *M* followed by a smaller, constant rate of growth for larger values of *M*. For the large *M* regime the maximum growth rate increases linearly with *M*. The cutoff and the most dangerous wavenumbers display a smaller rate of growth which scales with $M^{1/2}$. The cutoff wavenumber is approximately twice that of the most dangerous wavenumber, which is consistent with the analytical results of Yortsos and Hickernell[6]. This scaling analysis, based purely on numerical results, shows how the number of unstable waves will increase for larger values of the viscosity ratio and that the amplitude of the waves will increase at a higher rate.

Figure 3 plots the concentration contours of the injected fluid for *M*=100, Ca=100, and *A*=2. Viscous fingers are localized around the front while the expansion wave behind the front is stable. Notice that the gradually diminishes away from the tip. This variation of vorticity along the finger length, results in a pointed profile at the front, while a more dispersed profile is produced in the middle region or the finger root. The large concentration of equal amounts of positive and negative vorticity at the finger tips results in the absence of sideways motion which is necessary for coalescence and mechanical pinchoff mechanisms. Figure 6 shows, with the help of streamlines, that a strong shielding effect is present through which the smaller fingers disappear. The streamline pattern indicates how the stronger circulation generated by the larger neighboring fingers impedes the growth of the smaller fingers in the middle and lead to their eventual merging with the larger fingers.

3. Vertical Displacements with Density Variation

Two phase flow in porous media with gravity oriented in a direction parallel to the mean flow is of importance for enhanced oil recovery processes in petroleum reservoirs as well as for CO_2 sequestration in saline aquifers. The physical nature of the interaction of viscous and buoyancy related effects for such flows has not been previously investigated from the point of view of large-scale macroscopic instability.

We would therefore study the instability in vertical flow where the presence of shocks located at each end of an expansion wave represents a major challenge for a thorough analysis. Immiscible two-phase flow that results from the downward injection of a heavier fluid or upward injection of a lighter fluid, is characterized by two shocks, one at each end of a rarefaction wave. The specific details of the saturation profile, such as the shock speeds and the shock saturations, are determined by the fractional flow function for given values of the mobility ratio and the gravity number.

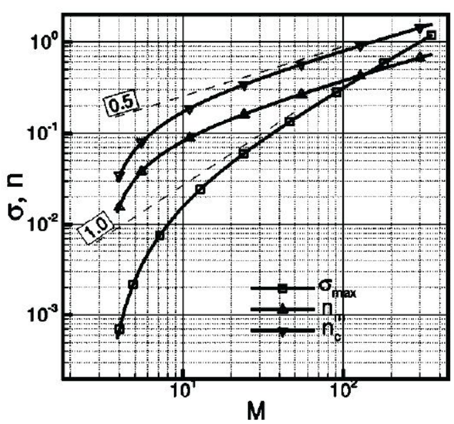

Figure 2. Maximum growth rate, the cutoff and the most dangerous wavenumbers as a function of the viscosity ratio.

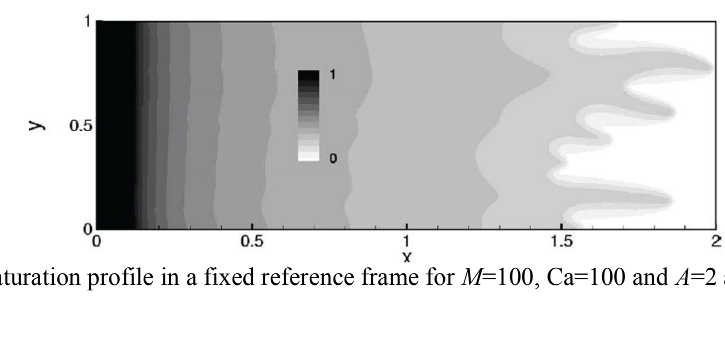

Figure 3. Full saturation profile in a fixed reference frame for *M*=100, Ca=100 and *A*=2 at *t*=0.2.

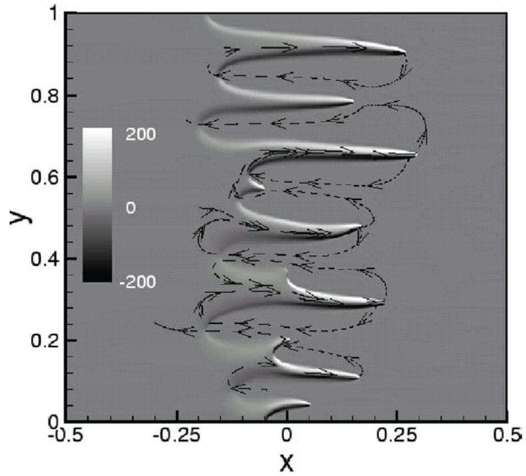

Figure 4. Vorticity contours and streamlines for M=50, Ca=200.

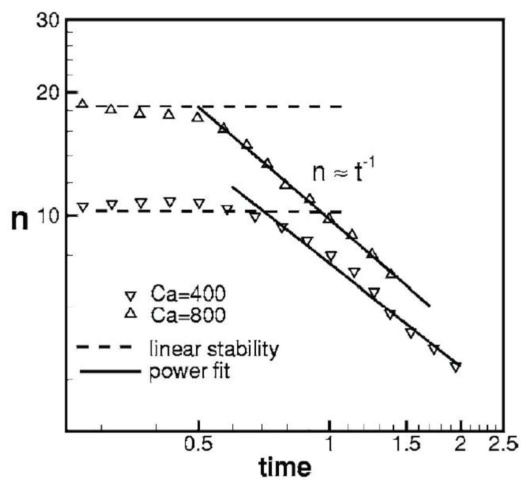

Figure 5. Evolution of the dominant mode n of nonlinear simulations for two capillary numbers.

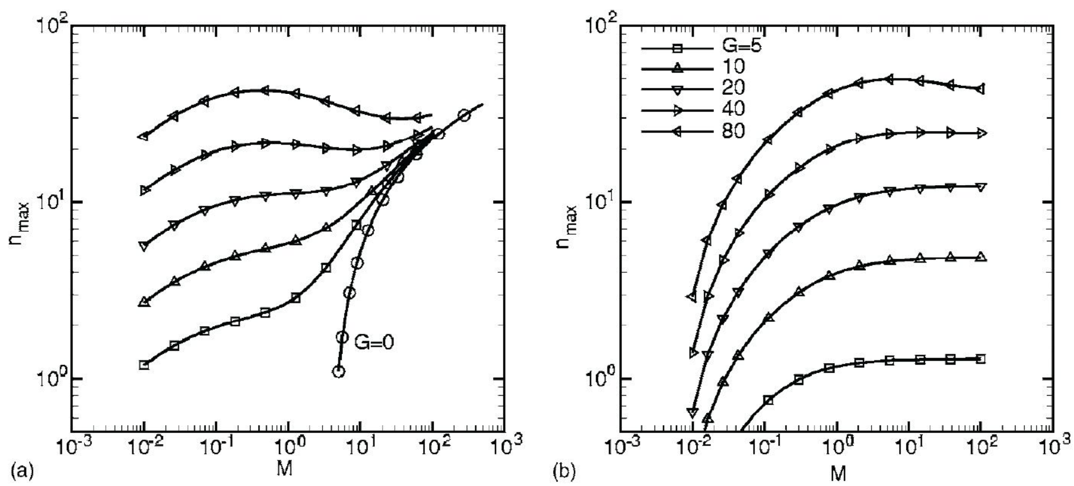

Figure 6. Most dangerous mode n_{max} as a function of M for different values of G, related to instability at the front end (a) and the back end (b).

The behavior of the most dangerous mode n_{max}_ is plotted in Figure 6 as a function of M for different values of G. Although a nonuniform growth of n_{max} as a function of M occurs only for larger values of G, flatter regions can be observed at intermediate values of M for small G. The influence of G becomes negligible and n_{max} tends to approach G=0 curve. The most dangerous mode at the back end increases uniformly with an increase in M, whereas one would have expected n_{max} to decrease uniformly in view of decreasing mobility gradients across the back end with an increase in M.

Figure 7 shows the structure of the unstable fingers due to gravitational and viscous instability for G=20, M=2 and at t=0.08. Unstable behavior can be observed at both the upward and downward moving fronts. Both the width and the length of the fingers are different at each end. The shock speeds however are similar as the fronts reach the respective boundaries at about the same time, even though the shock saturations are not the same. The unstable fingering structure is quite similar to that of the neutrally buoyant case, shown above. The mechanism of finger interaction and collapse, which is discussed in detail, appears to be relevant to this case.

Figures 8 and 9 show the influence of variation of the relative permeability function on the evolution of nonlinear unstable structures. These cases are for neutrally buoyant fluids and the injection is in the horizontal direction from left to right. The plotted contours span the range of saturations from 0 to 1. These figures highlight the importance of the relative permeability function in setting the wavelength and growth characteristics of unstable fingers. Comparison with the case shown in Figure 3 illustrates the difference between these cases with varying relative permeability functions. The standard relative permeability $k_{rn}=(1-S)^2$,for the case in Figure 3, leads to fingers that grow at a rate that is only slightly more than the speed of the forward moving shock. Therefore, the fingers only grow to relatively small amplitude by the time the front reaches the outlet boundary. Figures 8 and 9 on the other hand, are governed by the relative permeability functions, $k_{rn}=1-S^3$ and $krn=1-S^4$, respectively. These cases show a much larger influence of instability by the time the front comes close to the outlet boundary.

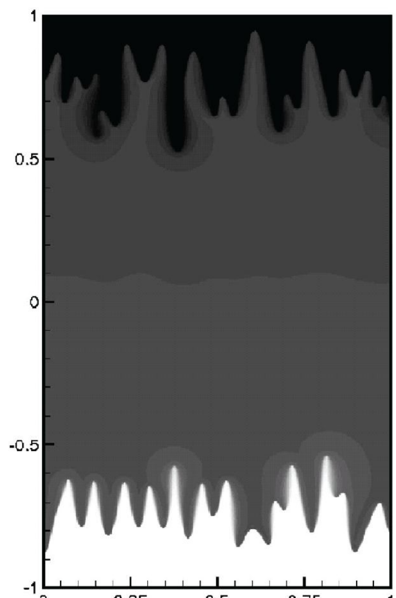

Figure 7. Saturation contours of an unstable vertical displacement.

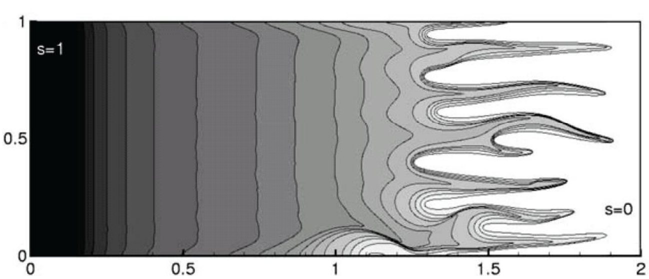

Figure 8. Saturation contours of an unstable horizontal displacement using a different relative permeability function.

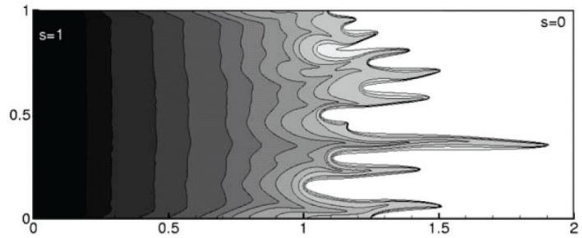

Figure 9. Saturation contours of an unstable horizontal displacement using a different relative permeability function.

4. Conclusions

The stability analysis of immiscible displacements shows that the ratio of the total mobility across the Buckley–Leverett shock front is the governing stability criterion. A ratio greater than unity leads to unstable flow, while stable behavior is predicted when this ratio is less then unity. While the shock mobility ratio is an appropriate criterion for the onset of instability it does not correlate well with either the maximum growth rate or the most dangerous and the cutoff wavenumbers. The maximum growth rate varies linearly with the viscosity ratio while the total-mobility ratio approaches an asymptotic value for large M. The most dangerous and the cutoff wavenumbers scale as $M^{1/2}$.

Immiscible two-phase vertical displacements in porous media give rise to shocks at each end of the rarefaction wave when gravity is destabilizing. We have applied the stability analysis to arbitrary mobility profiles to show that, in general, the maximum growth rate and the most dangerous mode at the front end become independent of the gravity number for large values of the mobility ratio. The most dangerous mode at the back end decreases as G is increased for large M. An independent treatment of individual fronts of the base saturation profile is justified in view of the results obtained from the numerical solution of the full problem. The growth rates and wavenumbers predicted by the theory are in reasonable agreement with numerical results.

According to the linear stability analysis, the maximum growth and the corresponding most dangerous mode are significantly influenced by the relative permeability functions. Our analysis determines how these quantities are influenced by with a change in the exponents of relative permeability functions. Accurate representation of the level of basic viscous instability in homogeneous displacements is an important step in designing improved analytical flow models. It is also important in analyzing the influence of permeability heterogeneity where the fundamental modes of viscous instability interact with the permeability length scales.

5. References

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Author Biographies

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