Critical Clearing Time for Wind Driven Generator

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Abstract

This paper presents an investigation on transient stability of power systems for a wind driven generator. The model considers the swing equations of two generators including a wind turbine generator in a power system while a fault happens at one of the transmission lines. Most stability studies model a wind turbine driven generator by a machine with small inertia that leads to instability of the machine during any fault. In this paper we considered that the machine is connected to a heavy flywheel to increase the stability during fault condition. The result of numerical simulation of the generators are presented and discussed.

1. Introduction

The stability of a generator or a system of interconnected dynamic components is its ability to return to normal or stable operation after being subjected to some form of disturbances. When the rotor of a synchronous generator advances beyond a certain critical angle, the magnetic coupling between the rotor and the stator fails [1-3]. The rotor, no longer held in synchronism with the rotating field of the stator currents, rotates relative to the field and pole slipping occurs. Each time the poles traverse the angular region where stability obtains, synchronizing forces attempt to pull the rotor into synchronism. When a fault occurs at the terminals of a synchronous generator or on a transmission line linked to it, the electrical power output of the generator is greatly reduced as it is supplying a mainly inductive circuit since generators internals and transmission lines are mainly inductive. However, the input power to the generator from the prime mover (wind turbine) practically cannot change immediately and even sometimes stays constant during the short period of the fault (from the time it starts until the time it is cleared) and the rotor tries to gain speed to store the excess energy that is being supplied from the prime mover. This energy is therefore is a stored energy. If the fault persists long enough the rotor angle will increase continuously and synchronism will be lost. Hence the time of operation of the protection and circuit breakers is all-important [4]. If the fault is cleared before the machine is put offline, the rotor starts to return (swing back) to its normal position. Due to its inertia and momentum, it may slip behind its initial position in the opposite direction. Then, if the magnetic coupling is still holding, it will try pulling the rotor back, and so on. Therefore, the movement of the rotor when assessing transient stability limits is studied over sufficient period of time and not just the first swing or up to the clearing time of fault [5-6].

2. Swing Equation

The power supplied by a generator k to a system comprising of N generators during a fault is described by the following so-called *Power Angle Equation*:

$$P_{ek} = \left|E_k'\right|^2 G_{kk} + \sum_{\substack{m=1 \\ m\neq k}}^{N} \left|E_k'\right|\left|E_m'\right|\left|Y_{km}\right| \cos(\delta_{km} - \theta_{km}) \tag{1}$$

where θ_{km} is the angle of the admittance Y_{km}, and $\delta_{km} = \delta_k - \delta_m$ is the rotor angle of generator k with reference to generator m. The Swing Equation of generator k is:

$$\frac{d^2\delta_k}{dt^2} = \frac{180f}{H_k} P_{ak} = \frac{180f}{H_k}(P_{mk} - P_{ek}) \tag{2}$$

where $d^2\delta_k/dt^2$ is the angular acceleration, H_k is the energy (inertia) constant of the generator which is the stored (kinetic) energy at synchronous speed per MVA rating of the generator, P_{mk} is the mechanical input from the prime mover to the generator, and P_{ak} is the accelerating power resulting from the difference between the mechanical and electrical powers.

The amount of acceleration and the direction of the swinging of the rotor angle during and after the transient period. From the swing equation, we clearly see that the acceleration of the rotor angle is affected by the inertia (or energy) constant H of the machine and the net accelerating power of $(P_m - P_e)$. The electrical power output P_e of the generator during the transient (for example fault) period is high for machines with low transient reactance, which can reduce the net acceleration and consequently improve the stability.

3. Potential and Kinetic Energy in Equal Area Criterion

Referring to Figure 1 that illustrates the equal area criterion, the kinetic energy is the energy stored in the rotating masses of the machine that is produced when the mechanical power output P_m from the prime mover to the generator is higher than the electrical power P_e supplied by the generator to the network. That is, the kinetic energy is the excess mechanical energy produced from the difference of $P_m - P_e$ (during the fault or during the disconnection of one of the transmission lines during normal operation) and indicated in Figure 1 by the shaded areaA_1. The potential energy is that produced due to the excess electrical power that may arise when a load is added, or when one of the transmission lines is disconnected due to operational requirement or by protection system due to fault. The potential energy is produced due to the de-acceleration when the electrical power P_e is higher than the mechanical power output P_m of the prime mover to the generator. It is indicated by the shaded areaA_2. The equivalent reactance between the machine internal voltage and the infinite bus is $X_d^{'} + X_{lines}$. The real power delivered by the synchronous generator to the infinite bus is:

$$P_e = \frac{\mathrm{E}^{'}\mathrm{V}_{\mathrm{bus}}}{\mathrm{X}_{\mathrm{d}}^{'} + \mathrm{X}_{\mathrm{lines}}} \sin\delta = P_{max} \sin\delta$$

When a fault occurs, P_e will greatly decrease since now no load is connected and the impedance is mainly the inductance of the generator internal transient reactance and the line (provided the fault at the line remote end from the generator). The curve noted as ‘Faulted’ is now the operating curve for the generator. At this moment, P_m is assumed the same as due to the dynamics of prime movers, it will take time until it reacts. The difference between P_m and P_e is the accelerating power which is a stored energy shaded indicated by the shaded areaA_1. Since P_m is higher, the rotor shaft will rotate faster than before, and therefore the rotor angle will increase in reference to the infinite bus. Though the generator still connected to the network through the other line, no considerable power is transferred to the network since the generator terminals are nearly earthed. When the fault is cleared at δ_{cr} , the generator will be operating on the ‘Postfault’ curve. The generator now transmits back the power to the network; however, it is less power than that before the fault since X_{lines} is increased. Due to the stored energy of area A_1, the acceleration of the rotor, and the inertia of the rotor, the rotor angle should continue to increase. Stability is lost when the rotor angle passes δ_{max} because then P_m would be greater than P_e again and rotor will accelerate and synchronism is lost. For the stability to remain, the energies indicated by A_1 and A_2 should be equal.

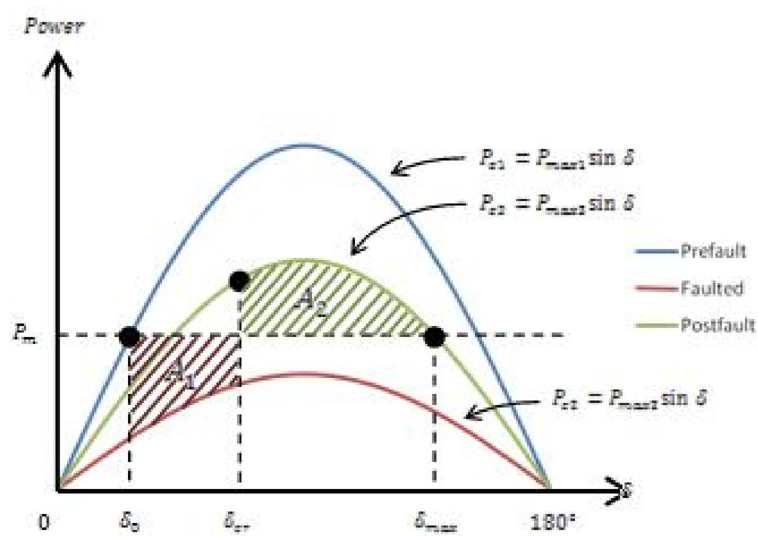

Figure 1. Power-Angle Curves for prefault, faulted, and postfault.

4. Case Study

The case is a model given in Reference [3]. A 50-Hz, 230-kV transmission system shown in Figure 2 has two generators of finite inertia and an infinite bus. The transformer and line data are given in Table 1. A three-phase fault occurs on line 4-5 near bus 4. Using the pre-fault power-flow solution Table 2, and the pre-fault and faulted bus admittance matrices given in Tables 3 and 4, determine the swing equation for Generator (1) and Generator (2) during the fault period. The generators have reactance and H values expressed on a 100-MVA base as follows:

Synchronous Generator (1):	400 MVA, 20 kV, X'_d=0.067 per unit,	H=11.2 MJ /MVA
Wind Generator (2):	250 MVA, 18 kV, X'_d=0.10 per unit,	H=8.0 MJ /MVA (with the flywheel)

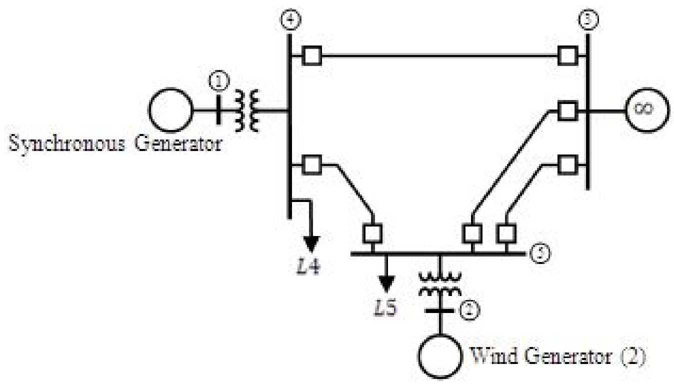

Figure 2. One-line diagram.

Table 1. Line and transformer data.*

Bus to bus	Series Z		Shunt Y
	R	X	B
Transformer 1-4	---	0.022	
Transformer 2-5	---	0.040	
Line 3-4	0.007	0.040	0.082
Line 3-5 (1)	0.008	0.047	0.098
Line 3-5 (2)	0.008	0.047	0.098
Line 4-5	0.018	0.110	0.226

*All values in per unit on 230-kV, 100-MVA base.

Table 2. Bus data and prefault load-flow values.*

Bus		Generation		Load	
		P	Q	P	Q
1	1.030/8.88°	3.500	0.712		
2	1.020/6.38°	1.850	0.298		
3	1.000/0°	---	---		
4	1.018/4.68°	---	---	1.00	0.44
5	1.011/2.27°	---	---	0.50	0.16

*Values are in per unit on 230-kV, 100-MVA base.

Table 3. Elements of prefault bus admittance matrix.*

Bus	1	2	3	4	5
1	-j11.2360	0.0	0.0		0.0
2	0.0	-j7.1429	0.0	0.0	J7.1429
3	0.0	0.0	11.2841-j65.4731	-4.2450+j24.2571	-7.0392+j41.3550
4	j11.2360	0.0	-4.2450+j24.2571	6.6588-j44.6175	-1.4488+j8.8538
5	0.0	J7.1429	-7.0392+j41.3550	-1.4488+j8.8538	8.9772-j57.2972

*Admittances in per unit.

Table 4. Elements of faulted and postfault bus admittance matrix.*

Bus	1	2	3
	Faulted network		
1	0.0000 – j11.2360 (11.2360/-90°)	0.0 + j0.0	0.0 + j0.0
2	0.0 + j0.0	0.1362 – j6.2737 (6.2752/-88.7563°)	-0.0681 + j5.1661 (5.1665/90.7552°)
3	0.0 + j0.0	-0.0681 + j5.1661 (5.1665/90.7552°)	5.7986 – j35.6299 (36.0987/-80.7564°)
	Postfault network		
1	0.5005 – j7.7897 (7.8058/-86.3237°)	0.0 + j0.0	-0.2216 + j7.6291 (7.6323/91.6638°)
2	0.0 + j0.0	0.1591 – j6.1168 (6.1189/-88.5101°)	-0.0901 + j6.0975 (6.0982/90.8466°)
3	-0.2216 + j7.6291 (7.6323/91.6638°)	-0.0901 + j6.0975 (6.0982/90.8466°)	1.3927 – j13.8728 (11.9426/-84.2672°)

In order to formulate the swing equations, we must first determine the transient internal voltages. Based on the data of Table 2, the current into network at bus 1 is:

$$I_1 = \frac{(P_1 + jQ_1)^*}{V_1^*} = \frac{350 - j0.712}{1.030\angle{-8.88^o}} = 3.468\angle{-2.619^o}$$

Similarly, the current into the network at bus 2 is:

$$I_2 = \frac{(P_2 + jQ_2)^*}{V_2^*} = \frac{1.850 - j0.298}{1.020\angle{-6.38^o}} = 1.837\angle{-2.771^o}$$

The transient internal voltage of a generator is calculated using the following equation:

$$E' = V_t + jX_d'I$$

where V_t is the generator terminal voltage and I is the output current. From this equation, we can calculate:

$$E_1' = 1.030\angle{-8.88^o} + j0.067 \times 3.468\angle{-2.619^o} = 1.100\angle 20.82^o$$

$$E_2' = 1.020\angle{-6.38^o} + j0.10 \times 1.837\angle{-2.771^o} = 1.065\angle 16.19^o$$

At the synchronous generator bus we have:

$$E_3' = E_3 = 1.000\angle 0.0^o$$

and so $\delta_{13} = \delta_1 \qquad \delta_{23} = \delta_2$

$$P_{e1} = |E_1'|^2 \underbrace{G_{11}}_{0} + |E_1'||E_2'|\underbrace{|Y_{12}|}_{0}\cos(\delta_{12} - \theta_{12}) + |E_1'||E_3'|\underbrace{|Y_{13}|}_{0}\cos(\delta_{13} - \theta_{13}) = 0$$

$$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_1'|\underbrace{|Y_{21}|}_{0}\cos(\delta_{21} - \theta_{21}) + |E_2'||E_3'||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= |E_2'|^2 G_{22} + |E_2'||E_3'||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2(0.1362) + (1.065)(1.0)(5.1665)\cos(\delta_2 - 90.755^o)$$

$$= 0.1545 + 5.5023\sin(\delta_2 - 0.755^o) \qquad \text{per unit}$$

Therefore, while the fault is on the system, the desired swing equations (values of P_{m1} and P_{m2} from Table 2) are:

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{H_1}P_{a1} = \frac{180f}{H_1}\left(P_{m1} - \underbrace{P_{e1}}_{0}\right) = \frac{180f}{11.2}(3.5) \qquad \text{elec deg / s}^2$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H_2}P_{a2} = \frac{180f}{H_2}(P_{m2} - P_{e2})$$

$$= \frac{180f}{8.0}\left\{\overbrace{1.85}^{P_m} - \left[\overbrace{0.1545}^{P_c} + \overbrace{5.5023}^{P_{max}}\sin\left(\delta_2 - \overbrace{0.755^o}^{\gamma}\right)\right]\right\}$$

$$= \frac{180f}{8.0}\left[\underbrace{1.6955}_{P_m - P_c} \quad \underbrace{5.5023}_{P_{max}}\sin\left(\delta_2 \quad \underbrace{0.755^o}_{\gamma}\right)\right]$$

$$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_1'|\underbrace{|Y_{21}|}_{0}\cos(\delta_{21} - \theta_{21}) + |E_2'||E_3'||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= |E_2'|^2 G_{22} + |E_2'||E_3'||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2(0.1591) + (1.065)(1.0)(6.0982)\cos(\delta_2 - 90.847^o)$$

$$= 0.1805 + 6.4946\sin(\delta_2 - 0.847^o)$$

The swing equation of machine 2 after the fault is:

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H_2}P_{a2} = \frac{180f}{H_2}(P_{m2} - P_{e2})$$

$$= \frac{180f}{8.0}[1.85 - 0.1805 - 6.4946\sin(\delta_2 - 0.847^o)]$$

$$= \frac{180f}{8.0}[1.6695 - 6.4946\sin(\delta_2 - 0.847^o)]$$

5. Steps for Constructing the Table and Plotting the Swing Curve

For simplicity, the number notation '2' for the machine number is eliminated where P_a and H refer to δ_2, P_{a2}, and H_2, respectively. The time interval is taken to be every 0.05 seconds starting from $t = 0$ when the fault occurred. The following is the step-by-step instruction to construct Table 5 (looking at the table elements (columns) from left to right).

for $t = 0$:

$\Delta\delta = 0$ since no change in the rotor angle yet happened until this moment. The angle is $\delta = 16.19^o$. The average of P_a is taken as $P_a/2$ since it was □ before the fault occurrence at $t = 0$- and increases immediately after the fault occurrence at $t = 0$+.

for all $t > 0$:

$$\Delta\delta_n = \Delta\delta_{n-1} + \left.\frac{d^2\delta}{dt^2}\right|_{\delta_{n-1}}$$

$$\delta_n = \Delta\delta_n + \delta_{n-1}$$

The swing curve is illustrated in Figure 3.

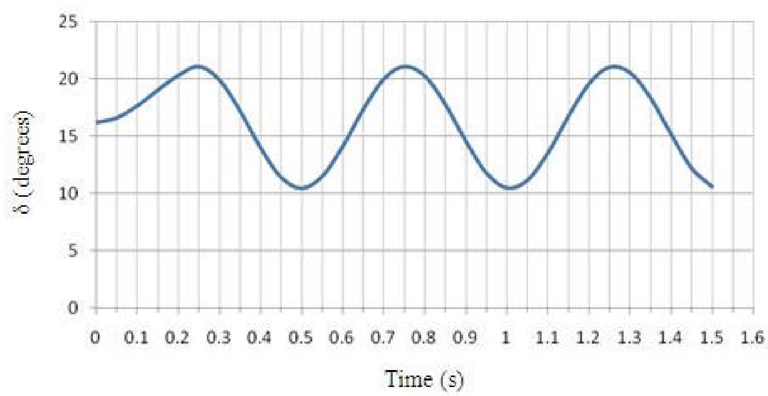

Figure 3. Swing curve for machine 2 for clearing time at 0.225 s.

6. Stability Analysis

The previous model is used to find out system's Transient Energy Function (TEF). It is well known from the literature that the loss of a transmission line will instantaneously decrease the transferable electric power capacity of remaining transmission system between generating units to load centers or infinite bus, whereas, mechanical input power to generating units remains constant at its pre-fault value for few seconds. This causes an imbalance between mechanical input power and transmitted electric power. This power imbalance during the fault causes the system's kinetic energy to increase that drives the generator towards an unstable region of operation (δ ≥ 90°). In Figure 1, the post-fault generator remains stable if extra energy stored in the generator during the fault is completely absorbed by transmission system. It is quite possible that post-fault power system may settle down to a new stable operating point provided the fault is cleared quickly and the post-fault power transmission capacity is restored to a desired level. On the other hand, a slow switching scheme employed for fault clearance may not prevent the generator to become unstable due to unmanageable energy imbalance. Therefore, it can be stated that the stability problem could become more acute if the transmission capacity is not restored to a required level in a post-fault power system. The post-fault maximum transmission capability (P_{max}) is very close to mechanical input power P_m, therefore, the fault has to be cleared very quickly to ensure that Area A1 stays equal to area A2. Actively control of the line reactance to raise or lower the transmission capacity is useful for contingency energy management leading to a proper balance of energy in a post-fault power system. To validate this claim Lyapunov's energy function (TEF) is chosen in this paper. The TEF is the time integral of swing equation of the test power system. The lower limit of the integration is the instant of the fault (t = 0, $\delta = \delta_0$), and the upper limit is the time instant when $\dot{\delta}$ becomes zero. The upper limit cannot exceed t_{max} that correspond to δ_{max} (= π-δ_0). The transient energy contained in the test power system is a time integral of accelerating power (Equation 3). Therefore, the time integral of the Lyapunov criteria TEF of the test power system as expressed below:

$$\text{TEF} = \int_0^{t_x} (M\ddot{\delta} + D\dot{\delta} + \frac{V_1 V_2 \sin(\delta)}{X_L} - P_m)dt \tag{3}$$

We solved to reveal properties contained in Equation 3 and used obtained solution for transient stability analysis. Equation 4 could express TEF of the test power system.

$$u(\delta,\ \dot{\delta}) = \frac{1}{X_L} = \frac{M\dot{\delta}^2 \sin(\delta) - P_m \cos(\delta) + D\dot{\delta}\cos(\delta) + w}{-V_1 V_2 \sin(\delta)\cos(\delta)} \tag{4}$$

Equation 4 is a variable quantity realized by the nonlinear controller and the SSSC system.

$$\text{TEF} = \int_0^{t_x} \{M\ddot{\delta} + D\dot{\delta} + \frac{M\dot{\delta}^2 \sin(\delta) - P_m \cos(\delta) + D\dot{\delta}\cos(\delta) - k\dot{\delta}\cos(\delta)}{-\sin(\delta)\cos(\delta)} \sin(\delta) - P_m\}dt \tag{5}$$

On following the steps given and substituting $w = -Mk\dot{\delta}\ \cos(\delta)$, we get the following equation:

$$\begin{aligned} \text{TEF} &= \int_0^{t_x} \left\{ M\frac{d}{dt}[\dot{\delta}\cos(\delta)] - Mk\dot{\delta}\cos(\delta) \right\} dt \\ &= M\dot{\delta}\cos(\delta(t_c)) - M\dot{\delta}\cos(\delta(t_0) - Mk\sin(\delta(t_c)) + Mk\sin(\delta(t_0)) \\ &= M\dot{\delta}\cos(\delta_c) - M\dot{\delta}\cos(\delta_0) - Mk\sin(\delta_c) + Mk\sin(\delta_0) \end{aligned} \tag{6}$$

In worst case scenario $\delta_c = \delta_{max}$ with $\delta_{max} > \delta_{cc}$. At $\delta = \delta_{max}$, ω $(= \dot{\delta} = d\delta/dt)$ becomes zero. Therefore, Equation 6 can be further simplified resulting into Equation 7 as follows:

$$\begin{aligned} \text{TEF} &= -Mk\sin(\delta_{\max}) + Mk\sin(\delta_0) \\ &= -Mk\sin(\pi - \delta_0) + Mk\sin(\delta_0) \\ &= \ 0 \end{aligned} \tag{7}$$

7. Conclusions

The authors presented a new method that can handle stability analysis of an electric power system with a wind turbine generator connected to an infinity bus system. Lyapunov criteria of stability were used to test the stability of the system. The result is promising for further analysis of electric power systems with wind turbine generators.

8. References

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